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Scattering and diffraction of sound by moving bodies

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A scattering configuration consists of a primary acoustic source S and a scattering body B, and we examine the effect of convection of S and B, at a uniform subsonic Mach number M, on the scattered field, for various types of source and scatterer. It is shown that if B is a compact rigid body scattering the near field of the source S it is *not* equivalent to a convected point dipole, but rather its pressure field is augmented by the quadrupole convection factor $(1-M\cos\theta)^{-3}$. If, on the other hand, the compact body B scatters the *distant* field of S, convective effects introduce an $O(M)$ monopole field which does not vanish in the side-line directions. A number of problems are examined in which B is a rigid half-plane, and there it is shown that the effect of convection is to augment the pressure by $(1-M\cos\theta)^{-\frac{3}{2}}$ in the case of diffraction of the field of a distant source S, and by $(1-M\cos\theta)^{-\frac{5}{2}}$ for scattering of the near field of S. Effects associated with the multipole order of S are discussed, as are those arising from the satisfaction of a Kutta condition at the trailing edge of the half-plane, and the application of these results to current problems in aerodynamic sound is mentioned.

1. Introduction

Much of the current effort, both theoretical and experimental, in aerodynamic noise is directed towards the study of mechanisms generating so-called 'excess noise'; i.e. measured engine noise levels in excess of those associated with the turbulent exhaust mixing process alone. Interest in these mechanisms is heightened by the fact that their fields change in a manner quite different from mixing noise under forward flight conditions. As a sweeping generalization one might say that mixing noise dominates most of the downstream arc, and there tends to be alleviated by forward flight, while excess noise dominates the side-line and forward-arc fields, and there tends either to remain more or less independent of flight speed or to increase in flight. A simple view consistent with this is that mixing noise is correlated with the shear across the mixing layer, and the shear drops in forward flight, while the excess noise is generated by sources within the engine, and if the sources are in some sense attached to the engine their fields will suffer Doppler amplification in the upstream arc in flight. In order to predict the noise of a given engine in flight one would therefore need to know (i) the levels and spectral distribution of mixing noise and excess noise under static conditions; (ii) the dynamic effects of flight on the strengths of the sources of mixing and excess noise; (iii) the acoustic effects due to convection of the sources attached to the engine. A great deal of work is at present being directed towards item (i), although this presents possible fundamental difficulties, in that an excess noise field might be undetectably weak statically and yet be so highly amplified by convection as to dominate the total flight noise field over some range of angles. Topic (ii) needs much further attention than it has so far received, though one might generally assume that the dynamic effects of flight are likely to be either beneficial (on mixing noise, for example) or negligible (on sources deep within an engine). Topic (iii) has also not received much attention in the past, and yet seems to offer possibilities for simple theoretical models which would at least indicate the nature and rough magnitude of effects to be expected. This paper aims to make a start in that direction by studying convection effects on one type of mechanism which is a potential source of excess noise: the scattering of intense near-field pressures by solid surfaces.

The results should have other aeronautical applications beyond the issue of excess engine noise. For example, the use of blown flaps as high-lift devices seems likely to set severe noise problems because of the sources associated with exhaust impingement on the flaps, and as the flap noise source is attached to the aircraft, one must anticipate a substantial increase in radiated flap noise ahead of the aircraft in flight. As another example, the use of aerodynamic surfaces as engine noise shields is likely to feature prominently in future designs, and it is necessary to know how the field in the shadow of a wing is changed by flight and how the interaction field between a wing trailing edge and an overwing shielded engine exhaust changes with flight conditions. If the noise sources can be adequately modelled, the results of this paper should be capable of predicting these changes, at least as far as the purely acoustic effects of flight are concerned.

Papers have, of course, been published previously on related theoretical lines. In particular, Cooke (1970), Jones (1972) and Candel (1972) have looked at variants of the half-plane problems with convection discussed in §5, though without the motivation we have here, while Ffowcs Williams & Hawkings (1969) have given a general formulation of the problem of sound generation by unsteady flow and surfaces in arbitrary convective motion. Their results, however, are completely formal, and consist essentially of integral restatements of the governing differential equation and boundary conditions, rather than explicit solutions of those equations for particular geometries. Consequently, one cannot be sure that the effects of convection are unambiguously revealed in such formal statements, and indeed we shall see (§4) that in some cases the formal solutions are misleading, while they are unable to offer any general prediction for the kind of problem discussed in §5.

We therefore examine below the details of some simple problems in which a source and a scattering body are convected along together at a subsonic Mach number M. For practical purposes we are often interested only in $M^2 \ll 1$, and the neglect of M^2 compared with unity offers a useful analytical simplification, while permitting purely acoustic convection effects which already show themselves at $O(M)$ to be distinguished from such $O(M^2)$ dynamical effects as the dependence of sound speed on convection velocity. We shall therefore always neglect M^2 compared with unity.

We start by giving some kinematic relations, and then recall the results for convected point multipoles in §3. We then find, in §4, that a compact rigid scattering body in motion is *not* equivalent to a convected point dipole, as one would naturally assume, but rather, suffers convective amplification of quadrupole order in the case of near-field scattering. When the compact body scatters the field of a distant source, convective effects cannot be represented entirely in terms of powers of the Doppler factor, as these effects turn out to introduce an $O(M)$ monopole field which, of course, survives in the side-line directions. This contradicts a widely held feeling among workers in aerodynamic noise, to the effect that convective effects must *vanish* in the side-line directions.

Next we examine the prototype non-compact inhomogeneous surface, the rigid half-plane, and find the effects of convection on the fields diffracted or scattered by the interaction of a source with the edge. Apart from the issue of trailing-edge Kutta conditions, dealt with by Jones (1972) and reproduced briefly in §5, it is found that convective effects here are always represented by Doppler factors of non-multipole order, lending support to the idea that edge scattering mechanisms can be regarded as some kind of fractional multipole. In the final section we summarize these results, give the orders of magnitude of the effects predicted in practical terms, and indicate problems on which further work is needed.

(An earlier version of this paper contained also a discussion of sound generation, rather than scattering, by moving bodies. Some of the results given there were erroneous, and as the generation aspect of the problem has in any case now been comprehensively treated in a forthcoming paper by Dowling (1975), the present paper accordingly restricts itself to scattering and diffraction problems.)

2. Kinematic relations

We are concerned with the kind of configuration shown in figure 1. A source S and a scattering body B are convected together in the x direction at a constant speed $U = a_0 M$, a_0 being the sound speed. S emits a signal at time t_S, which is scattered by B at time t_B and received by an observer at P at the current time t. We wish to compare this signal (characterized by the potential ϕ or pressure p) with that which would have been received at P in the static problem in which B has its position at time t_B and S its position at time t_S. In other words, the comparison is to be made at given values of (r, θ) and (R_0, Θ_0). If the distance r_0 is much less than a typical wavelength, it is equally relevant to make the flight: static comparison at given values of (r, θ) and (r_0, θ_0), as in this case the idea of a wave taking a well-defined time to get from S to B does not hold (though we are in no way suggesting that the results of the comparison will be the same at a given value of (r_0, θ_0) as at a given value of (R_0, Θ_0)).

As explained in §1, we limit ourselves to the case in which M^2 may be neglected *compared with unity*. Then the following relations hold:

$$\bar{r} = r(1 - M\cos\theta), \tag{2.1}$$

$$\cos\bar{\theta} = \frac{\cos\theta - M}{1 - M\cos\theta}, \quad \sin\bar{\theta} = \frac{\sin\theta}{1 - M\cos\theta}, \tag{2.2}$$

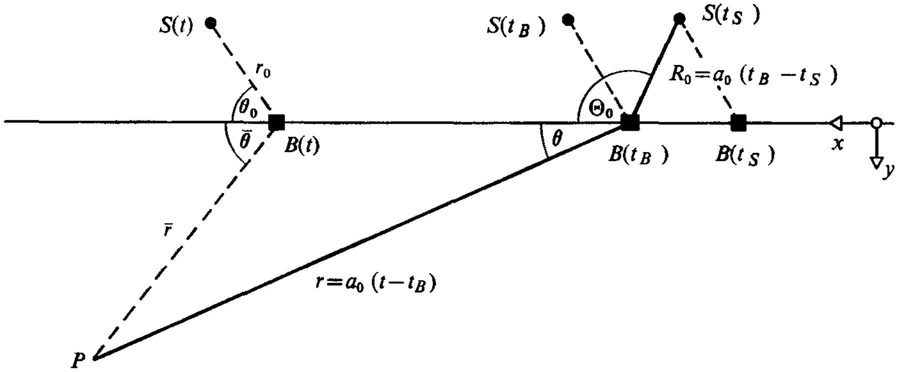

FIGURE 1. Configuration consisting of a primary source S and a scattering body B, convected together along the x axis. The diagram shows the position of a static observer P in relation to the positions of S and B at the time t_S of emission of a signal from S, at the time t_B of the scattering of that signal by B, and at the time t of reception of that signal at P. The signal heard at P is to be compared with that heard at P in the static problem in which S is located at $S(t_S)$ and B at $B(t_B)$.

$$r_0 = R_0(1 + M\cos\Theta_0), \tag{2.3}$$

$$\cos\theta_0 = \frac{\cos\Theta_0 + M}{1 + M\cos\Theta_0}, \quad \sin\theta_0 = \frac{\sin\Theta_0}{1 + M\cos\Theta_0}, \tag{2.4}$$

with an obvious difference in the sign of M between the two sets of relations.

The co-ordinate r is a function of x, y, z and t,

$$r^2 = [x - U(t - t_B) + Mr]^2 + y^2 + z^2, \tag{2.5}$$

and has the derivatives

$$\left(\frac{\partial r}{\partial x}\right)_t = \frac{\beta_1}{1 - M\cos\theta}, \quad \left(\frac{\partial r}{\partial y}\right)_t = \frac{\beta_2}{1 - M\cos\theta}, \quad \left(\frac{\partial r}{\partial z}\right)_t = \frac{\beta_3}{1 - M\cos\theta} \tag{2.6}$$

and

$$\dot r = -a_0 M\cos\theta/(1 - M\cos\theta), \tag{2.7}$$

$(\beta_1, \beta_2, \beta_3)$ being the direction cosines of the observer from the emission point $B(t_B)$.

For harmonic time dependence of the source, we shall find the distant field scattered by B to contain the phase factor $\exp ik(r - a_0 t)$, where r varies with t according to (2.5). If P is at rest relative to the fluid, the signal received is then not harmonic, and we can only *define* the observed frequency to be minus the time derivative of the phase. Thus

$$\omega(r,\theta) = -\frac{d}{dt}k(r - a_0 t) = \frac{\omega_0}{1 - M\cos\theta}, \tag{2.8}$$

where $\omega_0 = a_0 k$ is the static frequency. This is a general expression of the frequency shift known as the Doppler shift.

3. Sound generation by convected point multipoles

We now recall some well-established results for the fields radiated by *convected point multipoles* (see, for example, Morse & Ingard 1968, chap. 11). It will be shown in §4 that *none* of these results is identical with any of those for the fields scattered by small convected solid bodies.

An acoustic source is usually called a convected point monopole of strength $Q(t)$ if the pressure it generates satisfies the equation

$$\left(\nabla^2 - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right) p = -\frac{\partial}{\partial t}\{Q(t)\,\delta(x-Ut)\,\delta(y)\,\delta(z)\}, \tag{3.1}$$

the source being convected at speed U along the $+x$ axis and, for the moment, replacing the scattering body B. The corresponding potential satisfies the equation

$$\left(\nabla^2 - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right) \phi = \frac{1}{\rho} Q(t)\,\delta(x-Ut)\,\delta(y)\,\delta(z), \tag{3.2}$$

ρ denoting the mean fluid density. The free-field solution for the potential is

$$\phi = -\frac{1}{\rho}\frac{Q(t-r/a_0)}{4\pi r(1-M\cos\theta)}, \tag{3.3}$$

so that, using a prime to indicate differentiation with respect to the argument, the far-field pressure is, to $O(r^{-1})$, given by

$$p = \frac{Q'(t-r/a_0)}{4\pi r(1-M\cos\theta)^2}, \tag{3.4}$$

in view of (2.7). Note that, as in figure 1, the co-ordinates r and θ are time dependent, being measured from the source point at the time of emission.

A source is called a convected dipole of strength $\mathbf{F}(t)$ if it radiates a pressure field satisfying

$$\left(\nabla^2 - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right) p = \operatorname{div}\{\mathbf{F}(t)\,\delta(x-Ut)\,\delta(y)\,\delta(z)\}. \tag{3.5}$$

The solution can be found by taking spatial derivatives of (3.2) and (3.3) and using the relations (2.6). This yields

$$p = \frac{\beta_i F_i'(t-r/a_0)}{4\pi a_0 r(1-M\cos\theta)^2}, \tag{3.6}$$

so that, to $O(r^{-1})$,

$$\phi = -\frac{1}{\rho}\frac{\beta_i F_i(t-r/a_0)}{4\pi a_0 r(1-M\cos\theta)}. \tag{3.7}$$

Thus monopoles and dipoles concentrated at a point suffer the same effect under convection; for each, the potential acquires a factor $(1-M\cos\theta)^{-1}$ relative to its static value, while the pressure is increased by the factor

$$(1-M\cos\theta)^{-2}.$$

Why the effect of convection should be the same on a monopole as on a dipole is made more apparent if we write the monopole field (3.4) in the form

$$p = \frac{Q'(t-r/a_0)}{4\pi r(1-M\cos\theta)} + \frac{\cos\theta\, UQ'(t-r/a_0)}{4\pi a_0 r(1-M\cos\theta)^2}. \tag{3.8}$$

The first term represents the field of a convected 'simple source', generating mass but not momentum, with

$$\left(\nabla^2 - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right)p = -Q'(t)\,\delta(x-Ut)\,\delta(y)\,\delta(z), \tag{3.9}$$

while the second term represents the field from a convected point dipole, whose strength $\mathbf{U}Q(t)$ is the rate at which the convected monopole injects momentum into the field. It is this dipole contribution which leads to the appearance of $(1-M\cos\theta)^{-2}$ in the pressure fields of both convected point monopoles and dipoles. In contrast, the fields of a convected 'simple source' contain one less Doppler factor, the results for (3.9) being

$$p = \frac{Q'(t-r/a_0)}{4\pi r(1-M\cos\theta)}, \quad \phi = -\frac{1}{\rho}\frac{Q(t-r/a_0)}{4\pi r}. \tag{3.10), (3.11}$$

The results for multipoles of higher order are obvious. A point quadrupole $\mathbf{T}(t)$, for example, is associated with an equation

$$\left(\nabla^2 - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right)p = -\frac{\partial^2}{\partial x_i\partial x_j}\{T_{ij}(t)\,\delta(x-Ut)\,\delta(y)\,\delta(z)\}, \tag{3.12}$$

and with fields

$$p = -\frac{\beta_i\beta_j T''_{ij}(t-r/a_0)}{4\pi a_0^2 r(1-M\cos\theta)^3}, \tag{3.13}$$

$$\phi = \frac{1}{\rho}\frac{\beta_i\beta_j T'_{ij}(t-r/a_0)}{4\pi a_0^2 r(1-M\cos\theta)^2}. \tag{3.14}$$

4. Scattering by convected compact bodies

4.1. *Near-field scattering*

Here we examine the extent to which the results of §3 are relevant to the fields scattered by convected bodies of *small but finite* dimensions compared with the typical wavelength. For definiteness we consider a rigid sphere B of radius a, convected along at a Mach number M together with a source S, which we take to be an idealized monopole of strength $Q(t) = Q\exp(-ika_0t)$. Using $\overline{\psi}$ for the azimuthal angle about the convection axis we have $\bar{\mathbf{r}} = (\bar{r}\cos\bar{\theta}, \bar{r}\sin\bar{\theta}\cos\overline{\psi}, \bar{r}\sin\bar{\theta}\sin\overline{\psi})$ and $\mathbf{r}_0 = (r_0\cos\theta_0, r_0\sin\theta_0\cos\psi_0, r_0\sin\theta\sin\psi_0)$, and we write $\bar{\vartheta}$ for the angle between $\bar{\mathbf{r}}$ and $\mathbf{r}_0$, so that

$$\cos\bar{\vartheta} = \cos\bar{\theta}\cos\theta_0 + \sin\bar{\theta}\sin\theta_0\cos(\overline{\psi}-\psi_0).$$

We are concerned here with near-field scattering by a compact sphere, so that $ka \to 0$ with $r_0/a = O(1)$, and we calculate the scattered sound field using the simple, physically reasonable, kind of matching argument used, for example, by

Landau & Lifshitz (1959, p. 281). A convected wave field is found whose inner behaviour near the sphere agrees with the leading term of the outer behaviour of an incompressible flow around the body. After deriving the result we shall justify this procedure, as a strictly comparable argument for the far-field scattering discussed in §4.2 gives an incorrect result, even in the static case $M = 0$ – a problem which Landau & Lifshitz discuss using another method without mentioning the failure of the simple matching argument. The truth in fact is that the 'asymptotic matching principle' which should be used to relate the scattered wave field to the Laplace flow around the body is *only* a mathematical principle, and though the physical origin for it may seem plausible, that physical argument is not only unnecessary but may be misleading (see Crighton & Leppington 1973, appendix B). It will be seen in §4.2 that a more careful formal approach than that of Landau & Lifshitz is necessary to get the right results, even for such a simple problem as (effectively) that of plane-wave scattering by a static compact sphere.

In the case of near-field scattering, the motion is incompressible for $k\bar{r} \ll 1$, and the incident potential due to S behaves like $-(Q/4\pi\rho)\,|\bar{\mathbf{r}}-\mathbf{r}_0|^{-1}\exp(-ika_0t)$. The harmonic function ϕ^s which vanishes at infinity and which, when added to this incident field, has zero normal derivative on the sphere is

$$\phi^s = -\frac{Q\exp(-ika_0t)}{4\pi\rho}\sum_{n=0}^{\infty}\sum_{s=0}^{n}\left(\frac{n}{n+1}\right)\frac{a^{2n+1}}{(\bar{r}r_0)^{n+1}}\frac{\epsilon_s(n!)^2}{(n-s)!\,(n+s)!} \times P_n^{(s)}(\cos\bar{\theta})\,P_n^{(s)}(\cos\theta_0)\cos s(\bar{\psi}-\psi_0), \quad (4.1)$$

where ϵ_s is the Neumann symbol. For $\bar{r}/a \gg 1$ this reduces to

$$\phi^s \sim -\frac{Q}{4\pi\rho}\exp(-ika_0t)\frac{1}{2}\frac{a^3}{(\bar{r}r_0)^2}\cos\bar{\vartheta}, \quad (4.2)$$

and this is to be matched to the inner limit of a solution of the convected wave equation (which holds, to leading order, throughout the region $\bar{r} \gg a$). A solution with the required form is

$$\phi^s = \left(A\frac{\partial}{\partial\bar{x}}+B\frac{\partial}{\partial\bar{y}}+C\frac{\partial}{\partial\bar{z}}\right)\frac{\exp ik(\bar{r}+M\bar{x}-a_0t)}{\bar{r}} \quad (4.3)$$

provided that the inner limit

$$\phi^s \sim -\frac{\exp(ika_0t)}{\bar{r}^2}\left(A\frac{\bar{x}}{\bar{r}}+B\frac{\bar{y}}{\bar{r}}+C\frac{\bar{z}}{\bar{r}}\right) \quad (4.4)$$

agrees with (4.2), thus requiring

$$(A,B,C) = \frac{Q}{4\pi\rho}\frac{a^3}{2r_0^2}(\cos\theta_0, \sin\theta_0\cos\psi_0, \sin\theta_0\sin\psi_0). \quad (4.5)$$

Use of (4.5) in (4.3) then gives the far-field pressure scattered from B, referred to the $\mathbf{r}$ frame defined by the scatterer at its emission time, in the form

$$p^s = -\frac{Q}{8\pi}(ka)(ka_0)\left(\frac{a}{r_0}\right)^2\frac{\cos\vartheta}{(1-M\cos\theta)^3}\frac{\exp ik(r-a_0t)}{r}, \quad (4.6)$$

where ϑ is the angle between $\mathbf{r}$ and $\mathbf{r}_0$.

Apart from the factor $(1-M\cos\theta)^{-3}$, this expression is precisely the formula for the near-field scattering from a compact rigid sphere under *static* conditions (see, for example, Crighton & Leppington 1971, equation (2.9)), and thus we see that the static dipole field is in this case augmented by the *quadrupole* convection factor $(1-M\cos\theta)^{-3}$. This conclusion seems at variance with simple intuition, as exercised, for example, by Lowson (1965), who assumed the equivalence of small bodies and convected dipoles in an analysis of propellor noise, and by Ffowcs Williams & Hawkings (1969), who derived a formula which seemed to confirm Lowson's supposition. There is, however, no discrepancy, for, as shown by Dowling (1975), the Ffowcs Williams & Hawkings results do not completely expose convective effects in the way one might hope. The origin of the 'extra' Doppler factor can be seen from the fact that the enforcing of a boundary condition on the *potential* ϕ (which in the convective case is not simply proportional to p) leads naturally through (4.2) to the form (4.3), in which it is already clear that the scatterer is a *dipole of potential*, and therefore that the potential will be convectively amplified by $(1-M\cos\theta)^{-2}$, the pressure by $(1-M\cos\theta)^{-3}$.

It is evident that this result is quite independent of the nature of the source S, provided only that all sources involved fulfil the condition $kr_0 \ll 1$. For when $kr_0 \ll 1$ and $ka \ll 1$ the convection cannot change the potential incident on the sphere. It cannot change the amplitude of the incident potential, since we have neglected those changes in taking $M^2 \ll 1$, neither can it change the phase, for the phase changes over the whole scattering region are negligible. The only effect can be that of convection on the sound as it leaves the scattering region, and the magnitude of that effect is determined, regardless of the excitation from S, by the multipole order of the scatterer, and a compact rigid body scatters a dipole field of potential *regardless of its shape*. Thus the $(1-M\cos\theta)^{-3}$ factor of (4.6) applies quite generally to the scattering by any rigid compact three-dimensional body. In a purely two-dimensional situation, the corresponding factor is obviously $(1-M\cos\theta)^{-\frac{5}{2}}$.

One might expect these results to hold whatever the value of kr_0, for the inner scattered potential will always have a dipole form like (4.2) as $\bar{r}/a \to \infty$ and so will always 'match' (in the Landau & Lifshitz sense) an outer dipole field like (4.3). That, however, is an incorrect conclusion, for even in the static case the dipole field scattered from a nearby source ($kr_0 \ll 1$) gives way as kr_0 increases to a field in which there are comparable monopole and dipole elements, as will be seen in §4.2. The error lies in the matching scheme. One can in fact add to (4.3) an arbitrary monopole $D\bar{r}^{-1}\exp ik(\bar{r}+M\bar{x}-a_0t)$ and the outer field will still 'match' (4.2) in the sense of the asymptotic matching principle (as proved in various forms by Fraenkel 1969; Crighton & Leppington 1973). The monopole strength D can only be found from a higher-order matching involving the next inner approximation (cf. a comparable step in Crow's (1970) treatment of aerodynamic sound generation by matched expansions).

In §4.2 we carry out the higher-order matching for the case $kr_0 \to \infty$ in which D does not vanish. In the present case it is found that $D = 0$, and the reason why this might be expected comes from consideration of Howe's (1975) reverseflow theorem. This generalization of the usual reciprocal theorem allows the nearby source and distant observer to be interchanged subject to reversal of the mean flow, and then one solves for the near field scattered from an incident plane convected wave (cf. Crighton & Leppington 1971; Howe 1975). In this reciprocal problem the near field is clearly dominated by a dipole, so that in the original problem the far field must be dipole, and hence $D = 0$.

4.2. *Scattering of the field of a distant source*

As $r_0 \to \infty$, the potential incident on the sphere from the source S takes the plane-wave form

$$\phi^i \sim \exp\{-ik\bar{r}\cos\bar{\vartheta} + ikM\bar{r}\cos\bar{\theta}\} \tag{4.7}$$

times an overall multiplicative factor

$$-(Q/4\pi\rho r_0)\exp\{ikr_0(1-M\cos\theta_0) - ika_0t\}.$$

We seek an asymptotic approximation, as $ka \to 0$, to the scattered potential ϕ^s which satisfies the exact equation

$$\left[a_0^2 + \frac{\gamma-1}{2}U^2 - \frac{\gamma-1}{2}(\bar{\nabla}\phi^s)^2 - (\gamma-1)\frac{\partial\phi^s}{\partial t}\right]\bar{\nabla}^2\phi^s = \left(\frac{\partial}{\partial t} + \nabla\phi^s.\bar{\nabla}\right)\left(\frac{\partial\phi^s}{\partial t} + \tfrac{1}{2}(\bar{\nabla}\phi^s)^2\right) \tag{4.8}$$

for inviscid irrotational homentropic flow, and which makes

$$\partial(\phi^s + \phi^i)/\partial\bar{r} = 0 \quad \text{on} \quad \bar{r} = a. \tag{4.9}$$

The details of the solution to this problem, using a careful application of matched expansions, are given in the appendix, where, for simplicity, we consider only the axisymmetric case with $\sin\theta_0 = 0$. Equations (A 11) and (A 12) lead to the result for the scattered pressure

$$p^s = -\frac{iQ}{4\pi r_0}(ka_0)(ka)^2\frac{a}{r}\frac{\exp\{ikr + ikr_0(1-M\cos\theta_0) - ika_0t\}}{(1-M\cos\theta)^2}$$
$$\times\left\{-\tfrac{1}{3}(\cos\theta_0 - M)^2 + \tfrac{3}{4}M(\cos\theta_0 - M) - \frac{1}{2}\frac{(\cos\theta_0 - M)\cos\theta}{1-M\cos\theta}\right\}. \tag{4.10}$$

When $M = 0$ the directivity factor $-\frac{1}{3} - \frac{1}{2}\cos\theta\,\mathrm{sgn}(\cos\theta_0)$ agrees with established results (see, for example, Bowman, Senior & Uslenghi 1969, p. 376). Convective effects change the strength of the monopole by $O(M)$ and augment the monopole pressure by $(1-M\cos\theta)^{-2}$, while the dipole strength is also changed by $O(M)$ and the pressure of the dipole field augmented by $(1-M\cos\theta)^{-3}$. Again the quadrupole factor $(1-M\cos\theta)^{-3}$ gives the amplification of a dipole field, but in this case the coupling between monopoles and dipoles precludes a convective effect expressible purely in terms of powers of the Doppler factor. A most important point to notice about (4.10) is that the field is changed by $O(M)$ even at 90° to the convection direction, even in the axisymmetric case $\cos\theta_0 = \pm 1$. This contradicts a widely held (though of course unproven) idea among workers in aerodynamic noise: that, if one concentrates on the side-line direction, convection and velocity refraction effects are absent and one gets a true view of the source.

The comparison with the static field should really be made after the use of (2.3) and (2.4) to introduce the co-ordinates (R_0, Θ_0) in place of (r_0, θ_0), yielding

$$p^s = \frac{-iQk^2a^3a_0}{4\pi}\,\frac{\exp\{ikr+ikR_0-ika_0t\}}{R_0(1+M\cos\Theta_0)\,r(1-M\cos\theta)}$$
$$\times\left\{-\frac{1}{3}\,\frac{\cos^2\Theta_0}{(1+M\cos\Theta_0)^2(1-M\cos\theta)}+\tfrac{3}{4}M\,\frac{\cos\Theta_0}{(1+M\cos\Theta_0)\,(1-M\cos\theta)}\right.$$
$$\left.-\frac{1}{2}\,\frac{\cos\Theta_0\cos\theta}{(1+M\cos\Theta_0)\,(1-M\cos\theta)}\right\}. \tag{4.11}$$

This shows a rather complicated convective effect as far as the *source* Doppler factors are concerned. The differences between the powers of $1+M\cos\Theta_0$ that appear in the final bracket are subtle; $(1+M\cos\Theta_0)^{-1}$ appears in the final term because this dipole term is thrown up by the first-order inner solution, which in turn is generated by the linear term in $\bar{r}$ of the factor (4.7). The first term, the $O(1)$ monopole, is determined only by the *second*-order inner solution, and part of that depends upon the $\bar{r}^2$ term in the expansion of (4.7), hence involving a coefficient $(\cos\theta_0-M)^2 = \cos^2\Theta_0(1+M\cos\Theta_0)^{-2}$. The $O(M)$ middle term in the last factor in (4.11) also comes from the second inner solution, but this time from a particular integral involving the first-order solution linearly, and hence involving only the factor $\cos\theta_0-M = \cos\Theta_0(1+M\cos\Theta_0)^{-1}$. Such subtleties could hardly be anticipated, and therefore it is necessary to carry out calculations of this kind in some detail to see the variety of effects which may arise. Further complications presumably arise in the 'off-axis' case $\sin\Theta_0 \neq 0$, and no general conclusion can be drawn as to the dependence of the scattered field on the location of S. The multipole nature of S sets no problem, however. One merely multiplies (4.11) by whatever factor multiplies the new incident field from S (that factor being $\cos\Theta_0(1+M\cos\Theta_0)^{-1}$ for an axial dipole, for example).

5. Half-plane diffraction problems

In this section we discuss scattering and diffraction by non-compact inhomogeneous bodies. The simplest case, and certainly the simplest for generalization to include convective effects, involves the rigid semi-infinite plate, convected in its own plane. As in §4, we look at the case of a multipole source S convected along with the plate, both when $kr_0 \ll 1$ and $kr_0 \gg 1$. Further, the plate edge may be either a leading edge or a trailing edge. The latter opens up the possibility that we should enforce a Kutta condition on the unsteady trailing-edge flow by allowing vorticity to be shed from the plate.

Leaving that particular issue aside for the moment, we can see immediately what kind of results must emerge. Take the case of a source close to the plate edge, and consider the diffraction problem for the convected wave equation which arises in the $\bar{\mathbf{x}}$ frame attached to the plate. This problem can be reduced to the *same static problem* by a simple phase shift (which amounts to making a Lorentz transformation of the original problem involving still fluid and moving boundaries). Now the static problem is well known to have a solution incapable of any compact multipole interpretation (Crighton & Leppington 1971): in fact the potential is proportional to a fractional power of the wavenumber k, while the directivity is of the $\cos\frac{1}{2}\bar{\theta}$ variety. It is obvious then that when the transformation (2.1)–(2.2) is applied to this solution, Doppler factors raised to powers different from those associated with multipoles must appear. For example, the scattering of the field of a point monopole by the half-plane will be seen to involve factors like $(1-M\cos\theta)^{-\frac{3}{2}}$ and $(1-M\cos\theta)^{-\frac{5}{2}}$.

Thus, even leaving aside the question of effects due to the type and location of the source S, we see again that convective effects on the directivity of surface-scattered sound fields are *not universal*. For any compact rigid surface B scattering the near field of S convective effects on the pressure directivity are always accounted for by the $(1-M\cos\theta)^{-3}$ factor, regardless of the shape of the scatterer and the location and type of source S, as proved in §4. Inasmuch as any rigid scatterer is acoustically equivalent to a surface dipole distribution, one might expect the $(1-M\cos\theta)^{-3}$ factor to apply also to non-compact surfaces, though that is now seen to be an erroneous idea. In the case of a static half-plane, the elementary surface dipole contributions are so phased by the appropriate retarded times as to lead to a net scattered field not of multipole type. When the half-plane is convected the strengths of the surface dipole potentials are indeed unchanged in amplitude (if $M^2 \ll 1$), but their integrated effect is changed nonetheless because convection alters the retarded times at which the individual dipoles must emit, and there is no reason to expect this change to take a multipole form.

We indicate below the way in which results can be found for one particular configuration, and merely state the results for other cases. The direction of motion is again the $+x$ direction, and we consider the trailing-edge problem with the half-plane in $\bar{y}=0, \bar{x}>0$. (Note that, in *every case*, the angles θ, $\bar{\theta}$, θ_0 and Θ_0 are measured from the direction of motion, and so not necessarily from the half-plane.)

The half-plane is rigid, and is irradiated by a line monopole, so that the potential ϕ is independent of z and satisfies a convected Helmholtz equation

$$[\bar{\nabla}^2-(M\,\partial/\partial\bar{x}+ik)^2]\,\phi = \delta(\bar{x}-x_0)\,\delta(\bar{y}-y_0), \tag{5.1}$$

with the factor $(Q/\rho)\exp(-ik\,a_0 t)$ suppressed. The boundary condition on the plate is that

$$\partial\phi/\partial\bar{y} = 0 \quad (\bar{y}=0, \quad \bar{x}>0), \tag{5.2}$$

while $\partial\phi/\partial\bar{y}$ and the pressure $p = -\rho(\partial/\partial t - M\,a_0\,\partial/\partial\bar{x})\,\phi$ must be continuous across $\bar{y}=0, \bar{x}<0$. This does not necessarily entail the continuity of ϕ there (Jones 1972), though it does require that the jump in ϕ be constant in a frame moving with the mean flow, i.e.,

$$\phi(\bar{x},0+)-\phi(\bar{x},0-) = A\exp(-ik\bar{x}/M), \tag{5.3}$$

where A is arbitrary. Thus the unsteady tangential velocity may be discontinuous across the downstream extension of the plate, and the constant A, a measure of the strength of the oscillatory vortex sheet shed from the plate, may, if we please, be chosen to ensure satisfaction of a Kutta condition at the trailing edge.

To solve (5.1), write

$$\phi = \exp[ikM(\bar{x}-x_0)]\,u_0 + \exp(ikM\bar{x})\,u_A, \tag{5.4}$$

where

$$\left.\begin{aligned} (\nabla^2+k^2)\,u_0 &= \delta(\bar{x}-x_0)\,\delta(\bar{y}-y_0),\\ \partial u_0/\partial\bar{y} &= 0 \quad (\bar{y}=0, \bar{x}>0),\\ u_0(\bar{x},0+)-u_0(\bar{x},0-) &= 0 \quad (\bar{x}<0) \end{aligned}\right\} \tag{5.5}$$

and

$$\left.\begin{aligned} (\bar{\nabla}^2+k^2)\,u_A &= 0,\\ \partial u_A/\partial\bar{y} &= 0 \quad (\bar{y}=0, \bar{x}>0),\\ u_A(\bar{x},0+)-u_A(\bar{x},0-) &= A\exp(-ik\bar{x}/M) \quad (\bar{x}<0). \end{aligned}\right\} \tag{5.6}$$

We have again neglected M^2 compared with unity, though as Jones (1972), Candel (1972) and Cooke (1970) have shown, it is unnecessary to do this in these half-plane problems. There is not really any point in retaining M^2 here, however, as we were unable to retain it in §4, and as our aim in any case is to study purely acoustic effects due to convection. These are already present at $O(M)$, and it is useful to distinguish them from dynamical effects of compressibility, which do not occur until $O(M^2)$.

The problem for u_0 is just the classical static half-plane problem, while the eigensolution u_A can of course exist *only* if $A \neq 0$. Calculation of u_0 is a familiar matter (see, for example, Jones 1964, pp. 581–590; Bowman *et al.* 1969, chap. 8). For $k\bar{r} \ll 1$ and *general* values of kr_0 we find

$$u_0 \sim -\pi^{-1}(\bar{r}/r_0)^{\frac{1}{2}}\exp(ikr_0)\cos\tfrac{1}{2}\bar{\theta}\cos\tfrac{1}{2}\theta_0, \tag{5.7}$$

while the distant fields are given by

$$u_0 \sim \begin{cases} -\pi^{-1}(r_0/\bar{r})^{\frac{1}{2}}\exp ik\bar{r}\cos\tfrac{1}{2}\bar{\theta}\cos\tfrac{1}{2}\theta_0 \quad (k\bar{r}\gg 1, \quad kr_0 \ll 1), & (5.8)\\[2mm] -\dfrac{1}{2\pi}\dfrac{\exp i(k\bar{r}+kr_0-\frac{1}{2}\pi)}{k(\bar{r}r_0)^{\frac{1}{2}}}\dfrac{\cos\frac{1}{2}\bar{\theta}\cos\frac{1}{2}\theta_0}{\cos\bar{\theta}+\cos\theta_0} \quad (k\bar{r}\gg 1, \quad kr_0\gg 1), & (5.9)\end{cases}$$

provided, in the latter case, that $\bar{\theta}$ is not near to $\pm(\pi-\theta_0)$, the geometrical-optics boundaries. Calculation of u_A by Wiener–Hopf arguments is straightforward (Jones 1972), and leads to

$$u_A \sim -A\left(\frac{1+M}{M}\right)^{\frac{1}{2}}\exp\left(-\frac{\pi i}{4}\right)\left(\frac{k\bar{r}}{\pi}\right)^{\frac{1}{2}}\cos\frac{\bar{\theta}}{2} \tag{5.10}$$

at points so close to the edge that $k\bar{r} \ll M$, while for $k\bar{r} \gg 1$

$$u_A \sim \frac{A}{2}M^{\frac{1}{2}}(1+M)^{\frac{1}{2}}\frac{\exp i(k\bar{r}+\frac{1}{4}\pi)}{(\pi k\bar{r})^{\frac{1}{2}}}\frac{\cos\frac{1}{2}\bar{\theta}}{1+M\cos\bar{\theta}}. \tag{5.11}$$

The choice $A = 0$ excludes the possibility of vortex shedding from the plate. According to (5.7), the velocities at the edge will then have the mild $\bar{r}^{-\frac{1}{2}}$ singularities characteristic of edge diffraction fields in the absence of flow, and we say that in this case no Kutta condition prevails on the unsteady trailing-edge flow. In this case (but not, however, in the static half-plane problem) there are also $\bar{r}^{-\frac{1}{2}}$ singularities in the pressure and in the pressure jump across the plate, though these singularities are really so innocuous that they do not preclude the solution as physically unrealistic. In particular, the choice $A = 0$ must apply to all leading-edge problems, and must be expected to apply also at the trailing edge at sufficiently high frequencies. Just how high those frequencies must be is at present an entirely speculative issue, into which we shall not enter here.

In our trailing-edge problem the choice $A = 0$ leads to a distant-field potential

$$\phi_0 \sim \frac{1}{\pi}\left(\frac{r_0}{r}\right)^{\frac{1}{2}} \exp ikr \cos \tfrac{1}{2}\theta \cos \tfrac{1}{2}\theta_0 \frac{(1-M)^{\frac{1}{2}}}{1-M\cos\theta} \tag{5.12}$$

for the case $kr_0 \ll 1$. The potential is increased over its value in the static problem at the same values of $\mathbf{r}$ and $\mathbf{r}_0$ by the factor $(1-M)^{\frac{1}{2}}(1-M\cos\theta)^{-1}$ and the pressure is increased by $(1-M)^{\frac{1}{2}}(1-M\cos\theta)^{-2}$. Contrast this with the convective amplification factors $(1-M\cos\theta)^{-(n+\frac{1}{2})}$, for integer values of n, which apply to the fields of two-dimensional line multipoles, $n = 1$ corresponding to a monopole or dipole, $n = 2$ to a quadrupole, as far as amplification of the pressure is concerned. The Doppler factor and the directivity function appearing in (5.12) suggest an interpretation of near-field scattering by a half-plane as a fractional-order multipole, a $\frac{3}{2}$-pole, an interesting analogy which will be pursued in a later paper.

It is possible to eliminate the factor $(1-M)^{\frac{1}{2}}$, and make the result (5.12) rather neater, by making the comparison with the static problem at the same values of (R_0, Θ_0), instead of (r_0, θ_0). There is, however, no real reason for doing this when $kr_0 \ll 1$, as the edge of the plate is no more *the scattering centre* than any other point less than a wavelength from the edge. The result, for the record, is

$$\phi_0 \sim -\frac{1}{\pi}\left(\frac{R_0}{r}\right)^{\frac{1}{2}} \exp ikr \cos \tfrac{1}{2}\theta \cos \tfrac{1}{2}\Theta_0 \frac{1}{1-M\cos\theta}, \tag{5.13}$$

and is free of M dependence, except in the Doppler factor.

When $kr_0 \gg 1$ it is essential to make the transformation from (r_0, θ_0) to (R_0, Θ_0), and then the distant field, with $A = 0$, is found to be

$$\phi_0 \sim -\frac{1}{2\pi}\frac{\exp i(kr+kR_0-\frac{1}{2}\pi)}{k(rR_0)^{\frac{1}{2}}}\frac{\cos\frac{1}{2}\theta\cos\frac{1}{2}\Theta_0}{\cos\theta+\cos\Theta_0}. \tag{5.14}$$

This shows that the diffracted potential of a line monopole many wavelengths from the diffracting edge is entirely unchanged, up to $O(M)$, by convection. The pressure is, however, amplified by the factor $(1-M\cos\theta)^{-1}$, relative to its value in the static case.

Analogous results can be found for the leading-edge problem in which the plate $(\overline{y} = 0, \overline{x} < 0)$ moves in the $+x$ direction $(\theta = 0)$. For that problem *only* the choice $A = 0$ is relevant. Table 1 gives a summary of results for that case, and for the trailing-edge problem with $A \neq 0$, to which we now turn.

If vortex shedding from the plate is allowed, then any value of A except one gives a solution with mild velocity and pressure singularities at the edge. There is a unique value of A (Jones 1972),

$$A = -\frac{\exp i(kr_0 - Mkx_0 + \frac{1}{4}\pi)}{(\pi kr_0)^{\frac{1}{2}}}\cos\frac{\theta_0}{2}\left(\frac{M}{1+M}\right)^{\frac{1}{2}}, \tag{5.15}$$

which yields finite, and in fact vanishing, values of the acoustic pressure and velocity at the trailing edge. The field ϕ_A which must be added to ϕ_0 to ensure satisfaction of a Kutta condition in this way has the far-field form

$$\phi_A \sim -\frac{\exp i(kr+kR_0+\frac{1}{2}\pi)}{2\pi k(rR_0)^{\frac{1}{2}}}\cos\frac{\theta}{2}\cos\frac{\Theta_0}{2}M(1-M\cos\Theta_0), \tag{5.16}$$

which, however, is dominated by ϕ_0 for most values of kr_0, as consideration of (5.13) and (5.14) will show. Only when the source S lies within a 'hydrodynamic wavelength' U/ω of the edge does the eigensolution ϕ_A dominate. Then $kr_0 \ll M$, and the satisfaction of the Kutta condition causes an increase in the radiated potential by a factor of order M/kr_0. Whether or not the Kutta condition is satisfied, however, the field always has the $\cos\frac{1}{2}\theta$ directivity when $kr_0 \ll 1$.

These results can be extended to cover the effects of convection on the fields scattered or diffracted by the half-plane when the source S is a *point monopole.* Let $(\bar{r},\bar{\theta},\bar{z})$ be cylindrical co-ordinates in the convected $\bar{\mathbf{x}}$ frame and (r_0,θ_0,z_0) the fixed source co-ordinates in that frame. Then, leaving aside the question of Kutta conditions altogether, the potential satisfies

$$[\bar{\nabla}^2-(M\,\partial/\partial\bar{x}+ik)^2]\,\phi = \delta(\bar{x}-x_0)\,\delta(\bar{y}-y_0)\,\delta(\bar{z}-z_0)$$

and, with the aid of results for the static problem (Bowman *et al.* 1969, chap. 8), can be written as

$$\phi \sim \left(\frac{k}{8\pi^3R_1}\right)^{\frac{1}{2}}\frac{\exp i(kR_1+kM(\bar{x}-x_0)+\frac{1}{4}\pi)}{k(\bar{r}r_0)^{\frac{1}{2}}}\,\frac{\cos\frac{1}{2}\bar{\theta}\cos\frac{1}{2}\theta_0}{\cos\bar{\theta}+\cos\theta_0}, \tag{5.17}$$

where $R_1^2 = (\bar{r}+r_0)^2+(\bar{z}-z_0)^2$. This result holds for $k\bar{r} \gg 1$ and $kr_0 \gg 1$, and takes on a simple form only in the 'flyover plane' $\bar{z} = z_0$, to use an obvious expression. There, in the very distant field $\bar{r} \gg r_0$, we have

$$\phi \sim \left(\frac{k}{8\pi^3R_0}\right)^{\frac{1}{2}}\frac{\exp i(kr+kR_0+\frac{1}{4}\pi)}{(kr)(1-M\cos\theta)^{\frac{1}{2}}}\,\frac{\cos\frac{1}{2}\theta\cos\frac{1}{2}\Theta_0}{\cos\theta+\cos\Theta_0}. \tag{5.18}$$

Apart from a numerical factor, (5.18) is identical with the result (5.14) for a line source, except for the additional factor $(kr)^{\frac{1}{2}}(1-M\cos\theta)^{\frac{1}{2}}$ arising from spherical, rather than cylindrical, spreading. Thus when $kr_0 \gg 1$, the diffracted field from a point monopole is increased by $(1-M\cos\theta)^{-\frac{1}{2}}$ as far as the potential is concerned, while the pressure is amplified by $(1-M\cos\theta)^{-\frac{3}{2}}$. If $kr_0 \ll 1$ then the distant scattered field is

$$\phi \sim \left(\frac{k}{2\pi^3\bar{r}}\right)^{\frac{1}{2}}\exp\left[ik(\bar{r}+M\bar{x})+\tfrac{1}{2}\pi i\right]\left(\frac{r_0}{\bar{r}}\right)^{\frac{1}{2}}\cos\tfrac{1}{2}\bar{\theta}\cos\tfrac{1}{2}\theta_0 \tag{5.19}$$

in the flyover plane $\bar{z} = z_0$, and obviously here the potential will be increased over its static value by $(1-M\cos\theta)^{-\frac{3}{2}}$, the pressure by $(1-M\cos\theta)^{-\frac{5}{2}}$.

All these results can be generalized to permit the source S to be a multipole of arbitrary order, though, just as in §4, this aspect affects only the source Doppler factors $1+M\cos\Theta_0$ and not the dependence upon θ. For rigid half-planes the θ dependence is a characteristic only of whether the body is acting in a near-field scattering mode ($kr_0 \ll 1$) or a far-field diffracting mode ($kr_0 \gg 1$).

Configuration	Two dimensions	Three dimensions	Comparison at given
Trailing edge Kutta condition $kr_0 \ll M$	$\dfrac{M(1-M)^{\frac{1}{2}}}{2\pi k r_0}$	$\dfrac{M(1-M)^{\frac{1}{2}}}{2\pi k r_0(1-M\cos\theta)^{\frac{1}{2}}}$	r_0, θ_0
	$\dfrac{M(1-M\cos\Theta_0)}{2\pi k R_0}$	$\dfrac{M(1-M\cos\Theta_0)}{2\pi k R_0(1-M\cos\theta)^{\frac{1}{2}}}$	R_0, Θ_0
Trailing edge No Kutta condition $kr_0 \ll 1$	$\dfrac{(1-M)^{\frac{1}{2}}}{1-M\cos\theta}$	$\dfrac{(1-M)^{\frac{1}{2}}}{(1-M\cos\theta)^{\frac{3}{2}}}$	r_0, θ_0
	$\dfrac{1}{1-M\cos\theta}$	$\dfrac{1}{(1-M\cos\theta)^{\frac{3}{2}}}$	R_0, Θ_0
Trailing edge Kutta condition or no Kutta condition $kr_0 \gg 1$	1	$\dfrac{1}{(1-M\cos\theta)^{\frac{1}{2}}}$	R_0, Θ_0
Leading edge No Kutta condition $kr_0 \ll 1$	$\dfrac{(1+M)^{\frac{1}{2}}}{1-M\cos\theta}$	$\dfrac{(1+M)^{\frac{1}{2}}}{(1-M\cos\theta)^{\frac{3}{2}}}$	r_0, θ_0
	$\dfrac{1}{1-M\cos\theta}$	$\dfrac{1}{(1-M\cos\theta)^{\frac{3}{2}}}$	R_0, Θ_0
Leading edge No Kutta condition $kr_0 \gg 1$	1	$\dfrac{1}{(1-M\cos\theta)^{\frac{1}{2}}}$	R_0, Θ_0

TABLE 1. Ratio of *potentials* in convected and static cases for half-plane problems. The ratio of the pressures is obtained by multiplying each entry by $(1-M\cos\theta)^{-1}$. In each case, the primary excitation is due to a point monopole in three dimensions and to a line monopole in two dimensions. For the first two rows of entries the reference static potential is, of course, one on which no Kutta condition can be imposed.

Thus for purely two-dimensional problems the pressure field scattered ($kr_0 \ll 1$) by a rigid half-plane is increased by $(1-M\cos\theta)^{-2}$, while the diffracted ($kr_0 \gg 1$) pressure field is increased by $(1-M\cos\theta)^{-1}$. Of course, these Doppler factors represent *weaker* effects due to convection than the Doppler factor $(1-M\cos\theta)^{-\frac{5}{2}}$ (which is the analogue for two dimensions of the $(1-M\cos\theta)^{-3}$ factor obtained in §4) characterizing a convected compact body, and that is simply a reflexion of the fact that edge scattering by large surfaces is, statically, a more efficient process than scattering by compact bodies.

6. Conclusions

We have shown that convective effects on the sound fields scattered or diffracted by solid surfaces take a variety of forms, determined both by the nature of the surface and by the location and type of primary source. The effects do not seem to be predictable from elementary arguments, and to some extent seem to run counter to simple intuition. The most important results may be summarized as follows.

(i) For the near-field scattering by compact rigid bodies the convective amplification is *not* of dipole order, but rather quadrupole, with a pressure increase by $(1-M\cos\theta)^{-3}$. This can be viewed as arising from the fact that the boundary condition on the potential ϕ leads to an expression for ϕ of dipole form, so that ϕ suffers dipole convective amplification by $(1-M\cos\theta)^{-2}$ and p by $(1-M\cos\theta)^{-3}$. These are the only convective effects for the near-field scattering case.

(ii) No universal convective effect applies to the scattering by a compact body of the field of a distant source. The θ dependence cannot be expressed in terms purely of the Doppler factor $1-M\cos\theta$, and convective effects introduce $O(M)$ monopoles whose fields do not vanish at $\theta = 90°$. The dependence upon the position Θ_0 is also obscure.

(iii) For near-field scattering by rigid half-planes convective effects are represented by non-multipole Doppler factors, the pressure being increased by $(1-M\cos\theta)^{-\frac{5}{2}}$ in a three-dimensional problem, regardless of source type and location, and regardless of whether the edge is a leading or a trailing edge [though see (iv)].

(iv) The importance of the imposition of a Kutta condition on the unsteady flow at a trailing edge is confined to sources so close to the trailing edge that $kr_0 \ll M$.

(v) In the case of diffracted fields from distant sources, the location and type of primary source enter the convective effects. However, the θ dependence of the convective terms continues to involve only the nature of the scatterer, for example in the $(1-M\cos\theta)^{-\frac{3}{2}}$ factor which the pressure acquires when diffracted by a half-plane.

We should also note the emphasis placed upon the potential rather than pressure, as the fundamental field variable in these problems involving convection.

The predicted increases in mean-square pressure are quite significant in practical terms. With $M = 0{\cdot}2$ and $\theta = 30°$, a case of interest in an aircraft landing approach, for example, $\overline{p^2}$ is increased by about 3 dB in either of cases (i) or (iii) above (which should be contrasted with an anticipated flight reduction in mixing noise of, typically, 3–5 dB). This then is the kind of increase to be anticipated from blown-flap noise, which may fall under case (i) or (iii) depending upon the frequency of the unsteady flow which interacts with the flap, from exhaust and wing trailing-edge interaction noise, which is likely to occur with over-wind shielded engines, and possibly from obstruction noise sources of the kind described by Gordon (1969). These powerful sources of excess noise are associated with unsteady flow around obstacles within a jet pipe, and the compact limit is generally applicable, so that $\overline{p^2}$ would be expected to increase by $(1-M\cos\theta)^{-6}$ provided that the obstruction lies within a wavelength of the jet-pipe nozzle, enabling it to be aware of the convection. That, however, is no more than a plausible argument, and is unlikely to hold for those sources of internal noise, of great practical importance, at frequencies high enough that the sources are buried deep within the jet pipe. In such cases one might expect no convective amplification, at any rate as far as the total sound power delivered by the source is concerned, for the power is determined by the local environment of the source, and cannot be increased by remote effects such as convection of the jet pipe. However, this does not preclude the possibility of convective changes to the pressure, provided these ensure the constancy of the total radiated power. Problems to model this kind of situation involve different mean flows inside and outside a jet pipe and the attendant downstream shear layer, with excitation from sources within the jet pipe, and are very much more difficult than those considered here, though much in need of solution in view of the practical importance of noise sources within aeroengines.

A topic related to conclusion (ii) above concerns the calibration of transducers for the measurement of acoustic pressure fluctuations in a moving stream, a matter of some importance in view of widespread current attempts to use wind tunnels for simulation of the noise of aircraft in flight. The possible bearing of the kind of calculations presented here on the calibration issue is really a separate topic, which will be reported elsewhere.

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Appendix. Plane-wave scattering in the presence of mean flow

We use $\bar{r}$ here to denote the previous $\bar{r}$ made dimensionless with the wave-number k. Then as $\bar{r} \to 0$ the incident field is

$$\phi^i = \exp(-i\bar{r}\cos\bar{\vartheta} + iM\bar{r}\cos\bar{\theta})$$
$$\sim 1 - i\bar{r}\cos\bar{\theta}\,(\cos\theta_0 - M) - \tfrac{1}{2}\bar{r}^2\cos^2\bar{\theta}\,(\cos\theta_0 - M)^2 + \dots \qquad \text{(A 1)}$$

in the axisymmetric case, $\sin\theta_0 = 0$. Define an inner variable $\tilde{r} = \bar{r}/\epsilon$, $\epsilon = ka \to 0$; then the scattered potential satisfies the equation

$$\tilde{\nabla}^2\tilde{\phi} = (-i\epsilon + M\tilde{\nabla}\Phi_0 . \tilde{\nabla})(-i\epsilon + M\tilde{\nabla}\Phi_0 . \tilde{\nabla})\,\tilde{\phi}, \qquad \text{(A 2)}$$

where
$$\Phi_0 = -\cos\bar{\theta}(\tilde{r} + 1/2\tilde{r}^2)\,(1 + O(M^2)) \qquad \text{(A 3)}$$

is the potential of the mean flow. Assume an inner expansion

$$\tilde{\phi} = \epsilon\tilde{\phi}^{(0)} + \epsilon^2\tilde{\phi}^{(1)} + \dots; \qquad \text{(A 4)}$$

then
$$\tilde{\nabla}^2\tilde{\phi}^{(0)} = 0, \quad \tilde{\nabla}^2\tilde{\phi}^{(1)} = -2iM\tilde{\nabla}\Phi_0 . \tilde{\nabla}\tilde{\phi}^{(0)}, \qquad \text{(A 5)}$$

with, from (A 1),

$$\left.\begin{aligned} \partial\tilde{\phi}^{(0)}/\partial\tilde{r} &= i\cos\bar{\theta}\,(\cos\theta_0 - M) \\ \partial\tilde{\phi}^{(1)}/\partial\tilde{r} &= \cos^2\bar{\theta}\,(\cos\theta_0 - M)^2 \end{aligned}\right\} \quad \text{on } \tilde{r} = 1. \qquad \text{(A 6)}$$

Note that although the boundary values for $\tilde{\phi}^{(0)}$ and $\tilde{\phi}^{(1)}$ are just multiples of those for the case $M = 0$, the governing equations do not have the same property, so that the solutions for $M \neq 0$ cannot be found by scaling those for $M = 0$.

The inner solutions are

$$\tilde{\phi}^{(0)} = (-i/2\tilde{r}^2)\cos\tilde{\theta}\,(\cos\theta_0 - M), \tag{A 7}$$

$$\begin{aligned}\tilde{\phi}^{(1)} = M\,(\cos\theta_0 - M)\{(2\tilde{r})^{-1}\cos^2\tilde{\theta} + (4\tilde{r}^4)^{-1}\cos^2\tilde{\theta}\} \\ -(9\tilde{r}^3)^{-1}(3\cos^2\tilde{\theta}-1)\{(\cos\theta_0 - M)^2 + \tfrac{3}{4}M\,(\cos\theta_0 - M)\} \\ -(3\tilde{r})^{-1}\{(\cos\theta_0 - M)^2 + \tfrac{3}{4}M\,(\cos\theta_0 - M)\}\end{aligned} \tag{A 8}$$

plus, in each case, a general inner eigensolution. In each case it turns out that the eigensolutions are only needed at higher orders in ϵ.

The two-term inner solution $\epsilon\tilde{\phi}^{(0)} + \epsilon^2\tilde{\phi}^{(1)}$ is now written in terms of $\bar{r}$ and expanded to leading order in ϵ for $\bar{r} = O(1)$. That order is $O(\epsilon^3)$, and prompts an outer expansion

$$\phi \sim \epsilon^3\bar{\phi}^{(0)} + O(\epsilon^4), \tag{A 9}$$

where

$$\{\bar{\nabla}^2 - (M\,\partial/\partial\bar{x} + i)^2\}\,\bar{\phi}^{(0)} = 0, \tag{A 10}$$

with a general multipole solution

$$\bar{\phi}^{(0)} = \bar{A}\frac{\exp\{i(\bar{r}+M\bar{x})\}}{\bar{r}} + \bar{B}\frac{\partial}{\partial\bar{x}}\left(\frac{\exp\{i(\bar{r}+M\bar{x})\}}{\bar{r}}\right) + \bar{C}\frac{\partial}{\partial\bar{y}}\left(\frac{\exp\{i(\bar{r}+M\bar{x})\}}{\bar{r}}\right) + \bar{D}\frac{\partial}{\partial\bar{z}}\left(\frac{\exp\{i(\bar{r}+M\bar{x})\}}{\bar{r}}\right) + \dots. \tag{A 11}$$

Now we express $\epsilon^3\bar{\phi}^{(0)}$ in terms of $\bar{r}$, expand for $\tilde{r} = O(1)$, keeping terms $O(\epsilon)$ and $O(\epsilon^2)$, and then rewrite the result in terms of $\bar{r}$. Matching with the outer expansion of the inner two-term series gives

$$\left.\begin{aligned}\bar{B} &= \tfrac{1}{2}i\,(\cos\theta_0 - M),\\ \bar{A} &= -\tfrac{1}{3}(\cos\theta_0 - M)^2 + \tfrac{1}{4}M\,(\cos\theta_0 - M),\\ \bar{C} &= \bar{D} = \text{all higher coefficients} = 0.\end{aligned}\right\} \tag{A 12}$$

These results lead directly to the quoted equation (4.10) of the main text. The important point to note is that although the dipole strengths $\bar{B}$, $\bar{C}$ and $\bar{D}$ can be found from matching $\epsilon^3\bar{\phi}^{(0)}$ to $\epsilon\tilde{\phi}^{(0)}$ the monopole strength $\bar{A}$ remains undetermined, and can only be found by a higher-order matching with

$$\epsilon\tilde{\phi}^{(0)} + \epsilon^2\tilde{\phi}^{(1)}.$$

The naive matching of the leading dipole $\tilde{\phi}^{(0)}$ of the inner series with the inner asymptotics of the dipole elements in (A 11) omits a monopole just as important as the dipole.

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